

Strengthening Supercompilation for **Call-By-Value** Languages

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Background

Context

- Timber: a **pure** and higher-order **call-by-value** language
- Program **optimization** is our goal
- We care about **preserving semantics**

Wadler's algorithm

$T[\text{app } (\text{app } xs \ ys) \ zs] =$

$\text{app } xs \ ys = \text{case } xs \text{ of } \{ [] \rightarrow ys; (x:xs) \rightarrow x:\text{app } xs \ ys \}$

Wadler's algorithm

$T[\text{app } (\text{app } xs \ ys) \ zs] =$

$T[\text{case } \text{app } xs \ ys \ \text{of}$
 $[\] \rightarrow zs$
 $(x:xs) \rightarrow x:\text{app } xs \ zs]$

$\text{app } xs \ ys = \text{case } xs \ \text{of } \{[\] \rightarrow ys; (x:xs) \rightarrow x:\text{app } xs \ ys\}$

$$T[(\lambda x.y) e] = y$$

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$$e = \perp$$

$$\text{CBN: } (\lambda x.y) \perp \rightarrow y$$

$$\text{CBV: } (\lambda x.y) \perp \rightarrow (\lambda x.y) \perp \rightarrow \dots$$

$$T[(\lambda x.y) e] = y$$

$$e = \perp$$

Modify CBN algorithm to transform arguments first?

CBN: $(\lambda x.y) \perp \rightarrow y$

CBV: $(\lambda x.y) \perp \rightarrow (\lambda x.y) \perp \rightarrow \dots$

Naïve algorithm

$N[\text{app } (\text{app } xs \ ys) \ zs] =$

$\text{app } xs \ ys = \text{case } xs \text{ of } \{ [] \rightarrow ys; (x:xs) \rightarrow x:\text{app } xs \ ys \}$

Naïve algorithm

$N[\text{app } (\text{app } xs \ ys) \ zs] =$

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Naïve algorithm

$N[\text{app } (\text{app } xs \text{ } ys) \text{ } zs] =$

$N[\text{app } (\text{case } xs \text{ of } \{[] \rightarrow ys; (x:xs) \rightarrow x:\text{app } xs \text{ } ys\}) \text{ } zs] =$

$\text{case } xs \text{ of}$

$[] \rightarrow N[\text{app } (ys) \text{ } zs]$

$(x:xs) \rightarrow N[\text{app } (x:\text{app } xs \text{ } ys) \text{ } zs]$

$\text{app } xs \text{ } ys = \text{case } xs \text{ of } \{[] \rightarrow ys; (x:xs) \rightarrow x:\text{app } xs \text{ } ys\}$

```
app (app xs ys) zs
```

```
app (x:app xs ys) zs
```


```
app (app xs ys) zs
```



```
app (x:app xs ys) zs
```

$$\begin{aligned} N[\text{app} (\text{app } xs \ ys) \ zs] &= \text{case } xs \text{ of} \\ &\quad [] \rightarrow h1 \ ys \ zs \\ &\quad (x:xs) \rightarrow h1 (h2 \ x \ xs \ ys) \ zs \end{aligned}$$
$$\begin{aligned} h1 \ xs \ ys &= \text{case } xs \text{ of } \{ [] \rightarrow ys; (x:xs) \rightarrow x:h1 \ xs \ ys \} \\ h2 \ x \ xs \ ys &= x:\text{case } xs \text{ of } \{ [] \rightarrow ys; (x:xs) \rightarrow h2 \ x \ xs \ ys \} \end{aligned}$$

$N[\text{app} (\text{app } xs \ ys) \ zs] = \text{case } xs \text{ of}$
 $[] \rightarrow h1 \ ys \ zs$
 $(x:xs) \rightarrow h1 (h2 \ x \ xs \ ys) \ zs$



$h1 \ xs \ ys = \text{case } xs \text{ of } \{ [] \rightarrow ys; (x:xs) \rightarrow x:h1 \ xs \ ys \}$
 $h2 \ x \ xs \ ys = x:\text{case } xs \text{ of } \{ [] \rightarrow ys; (x:xs) \rightarrow h2 \ x \ xs \ ys \}$

Our algorithm

$D[\text{app } (\text{app } xs \ ys) \ zs] =$

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Our algorithm

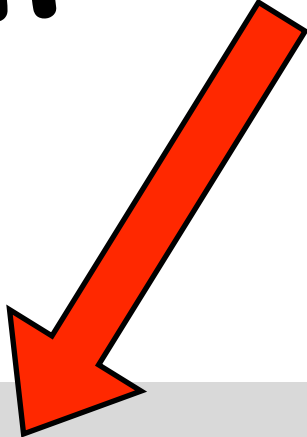
$D[\text{app } (\text{app } xs \ ys) \ zs] =$

$D[\text{let } xs = \text{app } xs \ ys, \ ys = zs \text{ in case } xs \text{ of}$
 $[\] \rightarrow ys$
 $(x:xs) \rightarrow x:\text{app } xs \ ys]$

$\text{app } xs \ ys = \text{case } xs \text{ of } \{[\] \rightarrow ys; (x:xs) \rightarrow x:\text{app } xs \ ys\}$

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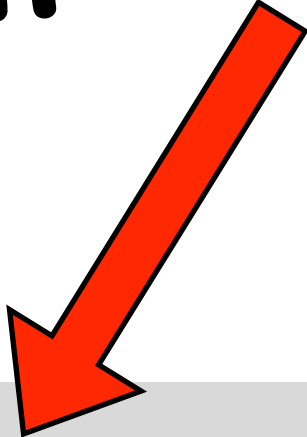
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$D[\text{app} (\text{app } xs \ ys) \ zs] = h3 \ xs \ ys \ zs$

$h3 \ xs \ ys \ zs = \text{case } xs \ \text{of}$

$\quad [] \rightarrow \text{case } ys \ \text{of}$

$\quad \quad [] \rightarrow zs$

$\quad \quad (y:ys) \rightarrow y:h4 \ ys \ zs$

$\quad (x:xs) \rightarrow x:h3 \ xs \ ys \ zs$

$h4 \ xs \ ys = \text{case } xs \ \text{of} \ \{ [] \rightarrow ys; (x:xs) \rightarrow x:h4 \ xs \ ys \}$

$D[\text{app} (\text{app } xs \text{ } ys) \text{ } zs] = h3 \text{ } xs \text{ } ys \text{ } zs$

$h3 \text{ } xs \text{ } ys \text{ } zs = \text{case } xs \text{ of}$

$[\] \rightarrow \text{case } ys \text{ of}$

$[\] \rightarrow zs$

$(y:ys) \rightarrow y \text{ } h4 \text{ } ys \text{ } zs$

$(x:xs) \rightarrow x:h3 \text{ } xs \text{ } ys \text{ } zs$

$h4 \text{ } xs \text{ } ys = \text{case } xs \text{ of } \{[\] \rightarrow ys; (x:xs) \rightarrow x:h4 \text{ } xs \text{ } ys\}$

(Almost) Follow Sørensen et al. (1996)

$E ::= [] \mid E \text{ es} \mid \text{case } E \text{ of alts}$

$$\dot{D}[x] = x$$

$$D[k \text{ es}] = k D[\text{es}]$$

$$D[E\langle(\backslash xs.e) \text{ es}\rangle] = D[E\langle\text{let } xs = \text{es in } e\rangle]$$

$$D[E\langle\text{let } x = e \text{ in } f\rangle] = D[E\langle[e/x]f\rangle] \text{ if } x \in \text{strict}(f) \\ \text{and } x \in \text{linear}(f) \\ = \text{let } x = D[e] \text{ in } D[E\langle f\rangle]$$

$$D[E\langle\text{case } x \text{ of } \{k_i x_i \rightarrow e_i\}\rangle]$$

$$= \text{case } x \text{ of } \{k_i x_i \rightarrow D[[k_i x_i/x]E\langle e_i\rangle]\}$$

$$D[E\langle\text{case } k_j \text{ es of } \{k_i x_i \rightarrow e_i\}\rangle] = D[E\langle\text{let } x_j = \text{es in } e_j\rangle]$$

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$$D[E\langle\text{case } k_j \text{ es of } \{k_i x_i \rightarrow e_i\}\rangle] = D[E\langle\text{let } x_j = \text{es in } e_j\rangle]$$

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$$D[\underline{E} \langle \underline{\backslash xs.e} \rangle \text{ es}] = D[\underline{E} \langle \text{let } xs = \text{es in } e \rangle]$$

$$D[\underline{E} \langle \text{let } x = e \text{ in } f \rangle] = D[\underline{E} \langle [e/x]f \rangle] \text{ if } x \in \text{strict}(f) \\ \text{and } x \in \text{linear}(f)$$

$$= \text{let } x = D[e] \text{ in } D[\underline{E} \langle f \rangle]$$

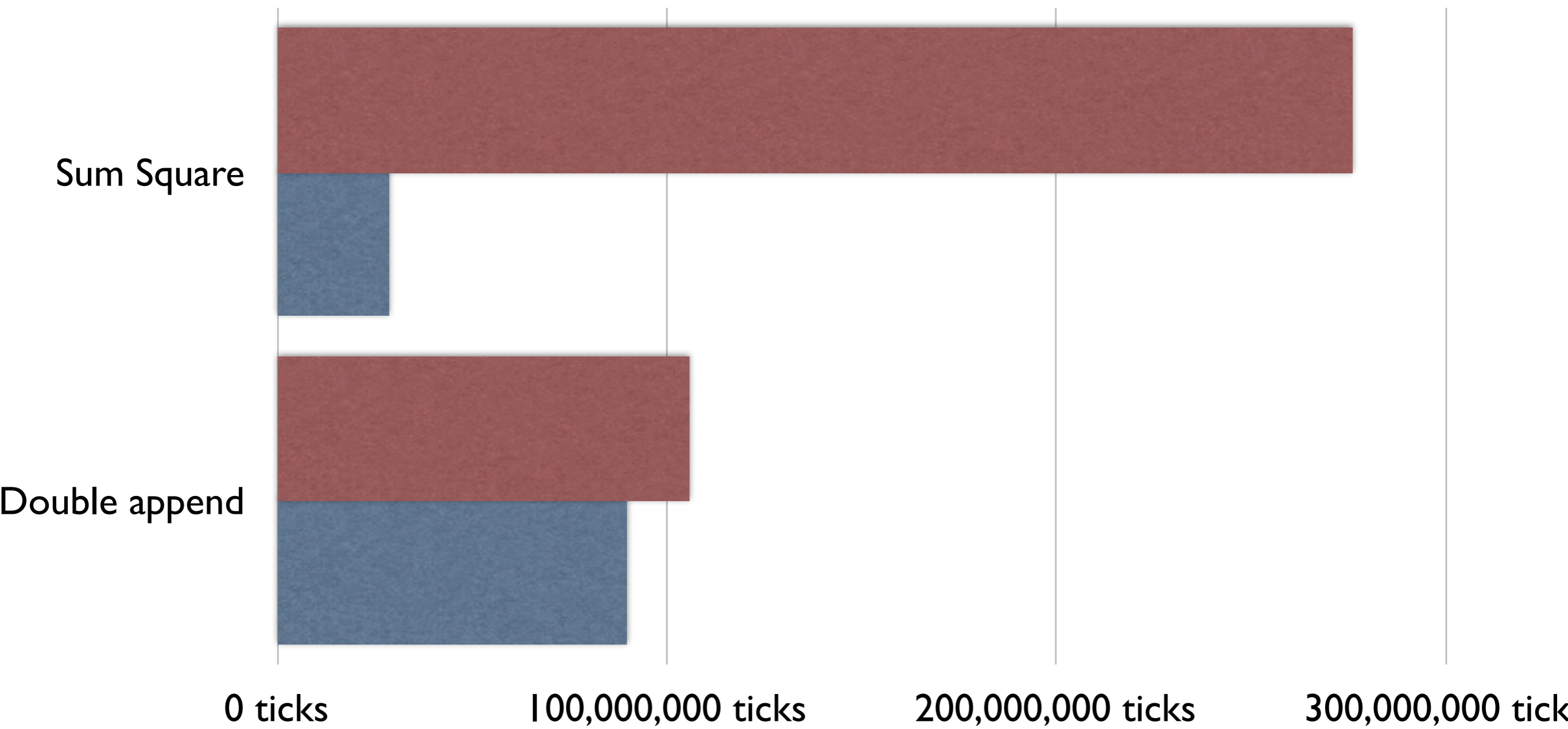
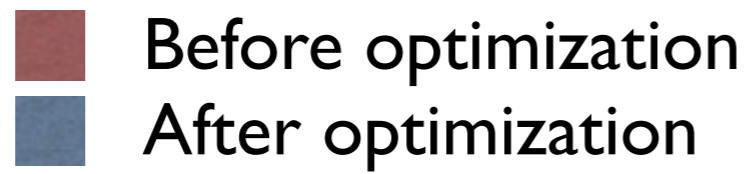
$$D[\underline{E} \langle \text{case } x \text{ of } \{k_i x_i \rightarrow e_i\} \rangle]$$

$$= \text{case } x \text{ of } \{k_i x_i \rightarrow D[[k_i x_i/x]E \langle e_i \rangle]\}$$

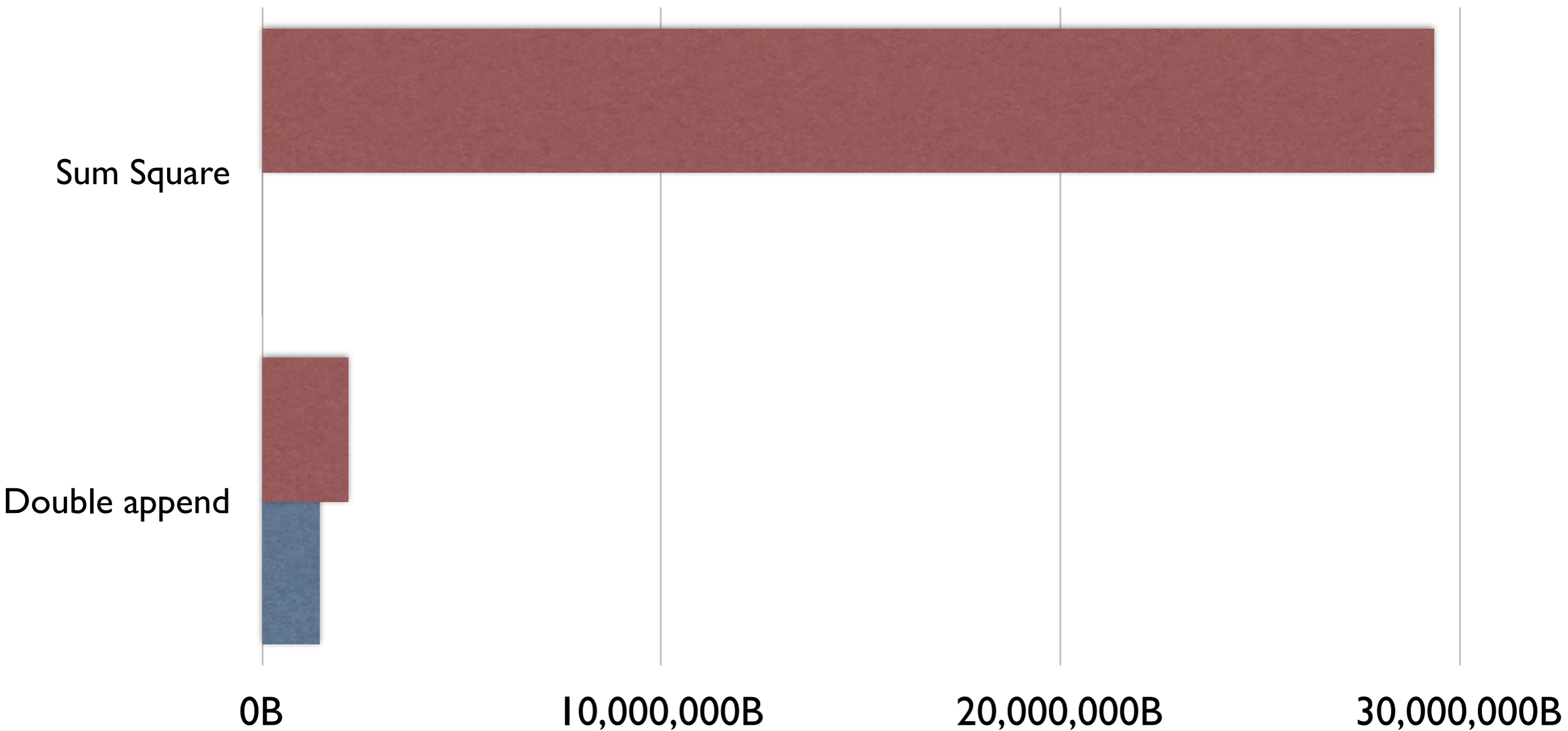
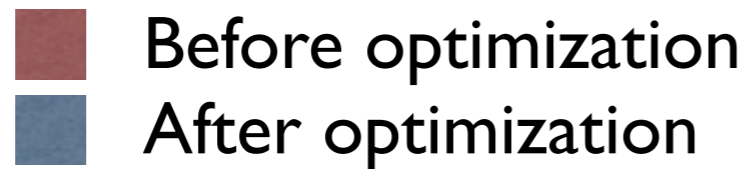
$$D[\underline{E} \langle \text{case } k_j \text{ es of } \{k_i x_i \rightarrow e_i\} \rangle] = D[\underline{E} \langle \text{let } x_j = \text{es in } e_j \rangle]$$



Runtime



Memory allocated



But..

**CBV-supercompilation is weaker
than CBN-supercompilation**

$D[\text{zip} (\text{map } f \text{ } xs') (\text{map } g \text{ } ys')] =$

CBV-supercompilation is weaker than CBN-supercompilation


$D[\text{zip } (\text{map } f \text{ } xs') \text{ } (\text{map } g \text{ } ys')] =$

$D[\text{let } ys = \text{map } g \text{ } ys'$
in $\text{case map } f \text{ } xs'$ of
 $[\] \rightarrow [\]$
 $(x':xs') \rightarrow \text{case } ys \text{ of}$
 $[\] \rightarrow [\]$
 $(y':ys') \rightarrow (x', y'):\text{zip } xs' \text{ } ys']$

CBV-supercompilation is weaker than CBN-supercompilation

$D[\text{zip } (\text{map } f \text{ } xs') \text{ } (\text{map } g \text{ } ys')] =$

$D[\text{let } ys = \text{map } g \text{ } ys'$
in $\text{case map } f \text{ } xs'$ of
 $[] \rightarrow []$
 $(x':xs') \rightarrow \text{case } ys \text{ of}$
 $[] \rightarrow []$
 $(y':ys') \rightarrow (x', y'):\text{zip } xs' \text{ } ys']$



Propagate Let-expressions

```
D[let ys = map g ys'
  in case (case xs' of
           [] -> []
           (x'' : xs'') -> f x'' : map f xs'') of
     [] -> []
     (x' : xs') -> case ys of
                   [] -> []
                   (y' : ys') -> (x', y') : zip xs' ys']
```

case xs of

[] -> D[let ys = map g ys'

in case [] of

[] -> []

(x':xs') -> case ys of

[] -> []

(y':ys') -> (x', y'):zip xs' ys']

(x'':xs'') -> D[let ys = map g ys'

in case f x'':map f xs'' of

[] -> []

(x':xs') -> case ys of

[] -> []

(y':ys') -> (x', y'):zip xs' ys']

$D[\text{zip} (\text{map } f \text{ } xs') (\text{map } g \text{ } ys')] = h5 \text{ } f \text{ } xs' \text{ } g \text{ } ys'$

$h5 \text{ } f \text{ } xs' \text{ } g \text{ } ys' =$

case xs' of

$[] \rightarrow \text{let } ys = \text{map } g \text{ } ys' \text{ in } []$

$(z:zs) \rightarrow \text{case } ys' \text{ of}$

$[] \rightarrow \text{let } x' = f \text{ } z, xs' = \text{map } f \text{ } zs \text{ in } []$

$(z':zs') \rightarrow (f \text{ } z, g \text{ } z') : h5 \text{ } f \text{ } zs \text{ } g \text{ } zs'$

The Case for Termination Analysis

```
h5 f xs' g ys' =  
  case xs' of  
    [] -> let ys = map g ys' in []  
    (z:zs) -> case ys' of  
      [] -> let x' = f z, xs' = map f zs in []  
      (z':zs') -> (f z, g z'):h5 f zs g zs'
```

Extended Let Rule

$D[E\langle \text{let } x = e \text{ in } f \rangle] B$

$= D[E\langle \text{let } B \text{ in } f \rangle] O$ if $\text{terminates}(e)$
and $x \notin \text{fv}(f)$

$= D[E\langle \text{let } B \text{ in } [e/x]f \rangle] O$ if $x \in \text{strict}(f)$
and $x \in \text{linear}(f)$

$= D[E\langle f \rangle] (B \cup (x, e))$

Future Work

- More measurements on real programs
- Investigate cheaper methods for ensuring termination
- What about accumulating parameters?

Related Work

- Supercompilers:
 - Scp4 (Nemytykh 2003)
 - Mitchell (2008, 2010)
 - Bolingbroke & Peyton Jones (2010)
 - Reich et al. (2010)

Conclusions

Supercompilation for call-by-value languages:

- can be strengthened to close in on call-by-name supercompilers