Supercompilation and the Reduceron

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“I wonder how popular Haskell needs to become for Intel to optimize their processors for my runtime, rather than the other way around.”

Simon Marlow, 2009
The Reduceron

- Special-purpose graph-reduction machine. (Naylor and Runciman, 2007 & 2010)
- Implemented on a Field Programmable Gate Array. (FPGA)
- Evaluates a lazy functional language;
  - Close to subsets of Haskell 98 and Clean.
  - Algebraic data types.
  - Uniform pattern matching by construction.
  - Local recursive variable bindings.
  - Primitive integer operations. ( +, −, =, ≤, ≠, emit, emitInt)
- Exploits low-level parallelism and wide memory channels in reductions.

See ICFP’10 paper “The Reduceron Reconfigured”.
Our source language

\[ \text{prog} := f \overline{\text{vs}} = \overline{x} \quad (\text{declarations}) \]

\[ \text{exp} := v \quad (\text{variables}) \]
\[ | c \quad (\text{constructors}) \]
\[ | \overline{f} \quad (\text{functions}) \]
\[ | f^P \quad (\text{primitive function}) \]
\[ | n \quad (\text{integers}) \]
\[ | \overline{x} \overline{xs} \quad (\text{applications}) \]
\[ | \text{case} \overline{x} \text{ of } \overline{c} \overline{\text{vs}} \rightarrow y \]
\[ | \text{let } \overline{v} \equiv \overline{x} \text{ in } y \]
An example

foldl \( f \) \( z \) \( xs \) = case \( xs \) of {
    Nil \quad \rightarrow \quad z;
    Cons \( y \) \( ys \) \quad \rightarrow \quad foldl \( f \) \( f \ z \ y \) \( ys \) ;;
}

map \( f \) \( xs \) = case \( xs \) of {
    Nil \quad \rightarrow \quad Nil;
    Cons \( y \) \( ys \) \quad \rightarrow \quad Cons \( f \ y \) \( \text{map} \( f \) \( ys \) \) ;;
}

\text{plus} \( x \) \( y \) = (+) \( x \) \( y \);
\text{sum} = \text{foldl} \ \text{plus} \ 0;

double \( x \) = (+) \( x \) \( x \);
\text{sumDouble} \( xs \) = \text{sum} \ (\text{map} \ \text{double} \ \text{xs});

range \( x \) \( y \) = case \( \leq \) \( x \) \( y \) of {
    True \quad \rightarrow \quad Cons \( x \) \ (\text{range} \ ((+) \ x \ 1) \ y);\n    False \quad \rightarrow \quad Nil \};

\text{main} = \text{emitInt} \ (\text{sumDouble} \ (\text{range} \ 0 \ 10000)) \ 0;
After case elimination

foldl f z xs = xs [foldl#1,foldl#2] f z;
foldl#1 y xs t f z = foldl f (f z y) xs;
foldl#2 t f z = z;

map f xs = xs [map#1,map#2] f;
map#1 y xs t f = Cons (f y) (map f xs);
map#2 t f = Nil;

plus x y = (+) x y;
sum = foldl plus 0;

double x = (+) x x;
sumDouble xs = sum (map double xs);

range x y = (≤) x y [range#1,range#2] x y;
ranged#1 t x y = Nil;
ranged#2 t x y = Cons x (range ((+) x 1) y);

main = emitInt (sumDouble (range 0 10000)) 0;
Reduction of an expression

range 0 10
Reduction of an expression

\[ \text{range 0 10} \]

\[ = \{ \text{Instantiate function body (1 cycle)} \} \]

\[ (\leq) 0 10 [\text{range#1,range#2}] 0 10 \]
Reduction of an expression

\[
\text{range } 0 \ 10
\]

\[
= \{ \text{Instantiate function body (1 cycle) } \}
\]

\[
(\leq) \ 0 \ 10 \ [\text{range#1,range#2}] \ 0 \ 10
\]

\[
= \{ \text{Primitive application (1 cycle) } \}
\]

\[
\text{True} \ [\text{range#1,range#2}] \ 0 \ 10
\]
Reduction of an expression

range 0 10

= \{ \text{Instantiate function body (1 cycle)} \} \\
(\leq) 0 10 [\text{range}\#1,\text{range}\#2] 0 10

= \{ \text{Primitive application (1 cycle)} \} \\
\text{True} [\text{range}\#1,\text{range}\#2] 0 10

= \{ \text{Constructor reduction (0 cycle)} \} \\
\text{range}\#2 [\text{range}\#1,\text{range}\#2] 0 10
Reduction of an expression

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\text{True} [\text{range#1,range#2}] 0 10
\]
\[
= \{ \text{Constructor reduction (0 cycle)} \} \\
\text{range#2} [\text{range#1,range#2}] 0 10
\]
\[
= \{ \text{Instantiate function body (2 cycles)} \} \\
\text{Cons} 0 (\text{range } ((+) 0 1) 10)
\]

Four cycles to reduce to HNF.
**Reduceron performance**

- The Reducer is running on a Xilinx Virtex-5 FPGA clocking at 96 MHz.
- Compare with an Intel Core 2 Duo E8400 clocking at 3 GHz.
- Sixteen benchmark programs.
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- Compare with an Intel Core 2 Duo E8400 clocking at 3 GHz.
- Sixteen benchmark programs.
- On average, 4.1x slower than GHC -O2.
- On average, 5.1x slower than Clean.
**Primitive reduct speculation**

\[
\text{range} \ 0 \ 10 \\
= \{ \text{Instantiate function body (1 cycle)} \} \\
(\leq) \ 0 \ 10 \ [\text{range#1, range#2}] \ 0 \ 10
\]
Primitive redex speculation

\[
\text{range } 0 \ 10 \\
= \{ \text{Instantiate function body (1 cycle)} \} \\
(\leq) \ 0 \ 10 \ [\text{range} #1, \text{range} #2] \ 0 \ 10
\]

- If tracing reduction by hand, you would evaluate the primitive.
- Why not the Reduceron?
- Primitive redex speculation (PRS) \( \text{currently} \) evaluates up to two primitives as the body is instantiated.
- Breaks laziness but as we are only dealing with reducible primitives, always terminates.
- Low cycle cost, often zero!
Reduction using PRS

range 0 10
Reduction using PRS

range 0 10

= \{ \text{Instantiate function body (1 cycle)} \} (\leq) 0 10 \ [\text{range#1},\text{range#2}] 0 10

= \{ \text{Primitive redex speculation (0 cycle)} \}
True \ [\text{range#1},\text{range#2}] 0 10
Reduction using PRS

\[
\text{range } 0 \ 10 \\
= \{ \text{Instantiate function body (1 cycle)} \} \\
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\text{True} [\text{range}\#1,\text{range}\#2] 0 10

= \{ \text{Constructor reduction (0 cycle)} \}
\text{range}\#2 [\text{range}\#1,\text{range}\#2] 0 10

= \{ \text{Instantiate function body (2 cycles)} \}
\text{Cons} 0 (\text{range} ((+) 0 1) 10)

= \{ \text{Primitive redex speculation (0 cycle)} \}
\text{Cons} 0 (\text{range} 1 10)

Three cycles to reduce further than HNF.
Performance using PRS

- Best speed-up — Queens by 2.4x.
- Taut has a marginal performance hit but is the only one.
- Nine out of nineteen examples see a speed-up of 1.1x or better.
Supercompilation

- A source-to-source compilation time optimisation
- Reduces the program as far as possible at compile-time.
- Where an unknown is required, proceeds by case analysis as far as possible.
- Can remove intermediate data structures and specialise higher-order functions.
- Our supercompiler is similar in design to that of Mitchell and Runciman. (2008)
**Supercompilation**

**Start**
- Tie Down the body of the main function

**Termination**
- Simple Termination?
  - Yes
    - For each child expression;
  - No

- Homeomorphic Termination?
  - Yes
    - Generalise the expression
  - No
    - Inline a saturated application

**Drive**
- Simplify the expression

**Tie**
- Tie Down and produce a fresh definition.
- Does an existing definition exist?
  - Yes
    - Tie Back to the existing definition
  - No

**Tie Children**
- For each child expression;

**Epilogue**
- Final Inlining with constant folding
- Dead Definition Removal
Drive

1. **Inline** the first saturated non-primitive application that does not cause driving to terminate. If all inlines cause termination, inline the first anyway.

2. **Simplify** the resulting expression using the twelve applicable simplifications listed in Peyton Jones and Santos (1994) and Mitchell and Runciman. (2008)
**Terminal Forms**

**Simple termination**

Terminate if expression is a;

- $v$ (free variable)
- $c$ (constructor)
- $n$ (integer)
- $v \overline{x}$ (app. to free)
- $f^P \overline{x}$ (prim. app.)
- case $v$ of $c \overline{vs} \rightarrow x$
- case $v \overline{x}$ of $c \overline{vs} \rightarrow x$
- case $f^P \overline{x}$ of $c \overline{vs} \rightarrow x$

**Homeomorphic termination**

Terminate if the expression homeomorphically embeds a previous derivation.

$$x \sqsubseteq y = dive \ x \ y \lor couple \ x \ y$$

$$dive \ x \ y = all \ ((\sqsubseteq) \ x) \ (children \ y)$$

$$couple \ x \ y = x \approx y \land and \ (zipWith \ (\sqsubseteq) \ (children \ x)(children \ y))$$
GENERALISATION

If a homeomorphic embedding is detected, attempt to \textit{generalise} the current expression.

1. If expressions are related by coupling, use \textit{most specific generalisation}. (Sørensen and Glück, 1995)

2. Otherwise, if the expression does not depend on any local bindings, \textit{lift the subexpression} that is coupled with the embedding. (Adapted from Mitchell and Runciman for a lambda-less language.)
If a homeomorphic embedding is detected, attempt to *generalise* the current expression.

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TIE

For each child expression;

1. **Tie back** *(fold)* — Where possible, replace the expression with an equivalent application of a previously derived definition.

2. **Tie down** *(residuate)* — Otherwise, replace the expression with an equivalent application of a newly produced definition and drive the new definition.
Performance using Supercompilation

- Best speed-up — Ordlist by 1.5x.
- Taut speeds up by 1.4x!
- Clausify gets marginally worse.
- Ten out of nineteen examples see a performance increase of more than 1.1%.
Performance through combined SC and PRS
Why does \texttt{sumDouble} do so well?

\begin{verbatim}
sumDouble supercompiled

h4 v v1 = case ((≤) v1 10000) of {
    False → v;
    True → h4 ((+) v ((+) v1 v1)) ((+) v1 1)};

main = emitInt (h4 6 3) 0
\end{verbatim}

- Gone from eight definitions to just two.
- Benefits from the removal of intermediate data structures.
- More PRS as the \texttt{foldl plus} expression has been specialised.
- Speed-up by a factor of 5.8x!
Why is Queens disappointing?

- Speed-up factor of 2.38x under PRS.
- Only 2.04x under SC+PRS.
- Supercompiler splits primitive redexes across case alternatives.
- The original program evaluated some primitives speculatively and in parallel.
- Supercompiled program does not utilise this feature.
- Not a one off, can happen to any program. Just particularly noticeable in Queens.
**Primitive Lifting**

- PRS can evaluate up two primitive redexes for free with each Reduceron body instantiation.
- Reduceron bodies map to source language;
  1. Function definitions.
  2. Case alternatives.
- Move the primitive redexes to maximise utilisation of this feature.
- Extract things that are potential primitive redexes as let-bindings.
- Lift the binding to the highest valid body root that has spare capacity, prioritising the expressions coming through less case distinctions.
RETURN TO `sumDouble`

\[
h_4 \ v \ v_1 = \text{case } ((\leq) \ v_1 \ 10000) \text{ of } \{
\begin{align*}
\text{False} & \rightarrow v; \\
\text{True} & \rightarrow h_4 \ ((+ \ v \ ((+ \ v_1 \ v_1))) \ ((+ \ v_1 \ 1)) \} ;
\end{align*}
\]
RETURN TO sumDouble

\[ h4\ v\ v1 = \text{case}\ (\leq\ v1\ 10000)\ \text{of}\ \{ \]
\[ \text{False} \rightarrow v; \]
\[ \text{True} \rightarrow h4\ ((+)\ v\ ((+)\ v1\ v1))\ ((+)\ v1\ 1) \}; \]

\[ h4\ v\ v1 = \text{let}\ \{ \]
\[ \text{prs} = (+)\ v1\ v1; \]
\[ \text{prs1} = (\leq)\ v1\ 10000 \]
\[ \} \text{in}\ (\text{case}\ \text{prs1}\ \text{of}\ \{ \]
\[ \text{False} \rightarrow v; \]
\[ \text{True} \rightarrow \text{let}\ \{ \]
\[ \text{prs2} = (+)\ v1\ 1; \]
\[ \text{prs3} = (+)\ v\ \text{prs} \]
\[ \} \text{in}\ (h4\ \text{prs3}\ \text{prs2}) \]
Laziness vs. Speculation

- Supercompilation simplifications are permitted to duplicate code as long as they do not duplicate computation. e.g. Let-bindings down case alternatives.
- Lifting primitive expressions will bring the duplicate code above case distinctions.
- Doesn’t matter under lazy evaluation.
- Wastes resources under speculative evaluation.
- **Solution:** Merge duplicate expressions into a single binding.
Performance using PRS, SC and Lifting
Summary

- Primitive-heavy programs can benefit from PRS.
- Supercompilation can speed up programs by removing intermediate data structures and specialising higher-order functions.
- Supercompilation aids PRS by making primitive redexes apparent where they were not previously.
- Further transformation is required to maximise utility of PRS.
- Results in an average combined speed-up by $1.7x$.
- With SC, PRS and lifting, the Reduceron is now only $2.5x$ slower than GHC -O2 on Intel.
Conclusions

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- If we rethink our execution, we have to rethink our optimisations.
- PRS and Supercompilation are not just complementary but synergistic.
- Must always ensure that we consider execution model when developing transformations.
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- Further investigation of disappointing examples.
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