A Simple Supercompiler Formally Verified in Coq

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Outline

1. Introduction
   - Questions on the Title
   - Decomposition of Supercompilation
   - Coq Features Used
2. Supercompiler Organization and Correctness Proof
   - Expression Language and Simple Normalization
   - Propagation of Test Outcomes in Branches
   - Full Language, Loop Unrolling
3. Possible Extensions and Applications
   - Test Generation, Extensional Equivalence
   - More Realistic Language
   - Use Information Propagation in Isolation

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- Supercompiler?
- Formal verification?
  - Important for non-experimental supercompilers
  - Fresh look over supercompilation process
- In Coq?
  - A matter of taste
  - Non-critical (very few Coq-specific features used)
- Simple?
  - Toy language ...
  - ... over a toy data domain (simple binary trees).
  - Cut supercompilation into smaller pieces ...
  - ... with modular proofs of correctness.
  - But: less powerful supercompiler

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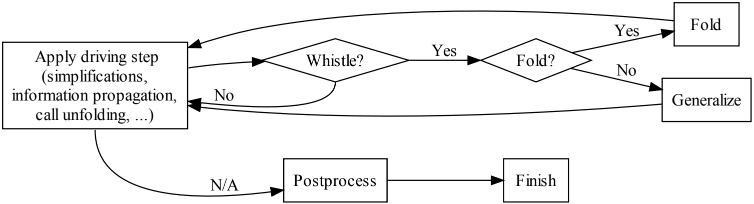
Decomposition of Supercompilation (classical).

Classical Organization of Supercompilation



Decomposition of Supercompilation (this work).

- Simple normalization (≈ deforestation – unfolding) - `normConv`

```
Example term := IfNil Id Id (Tl # Hd).
Eval compute in (ntrm2trm (normConv (term $ term))).
     = IfNil Id (IfNil Id Id (Tl # Hd)) (Hd # Tl) : Trm
Theorem normConvPreservesEval: forall (t: Trm) (v: Val),
    evalNT (normConv t) v = evalT t v.
```

- Propagation of test outcomes inside if-branches - `norm`

```
Eval compute in (ntrm2trm (norm (term $ term))).
     = IfNil Id Nil (Hd # Tl) : Trm
Theorem normPreservesEval: forall t v,
    evalNT (norm t) v = evalT t v.
```

- Single-step loop unrolling - `unrollToInit`
- Ensuring termination - `firstEmbedded`
- Multi-step loop unrolling - `sscp`

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Coq Features Used.

- Coq: Proof assistant based on CoC + inductive types
- Simplified point of view:
  - Total(!) functional programming language
    - Inductive datatypes (`Inductive ... := ... | ... .`)
    - Pattern matching (`match ... with ... => ... | ... end`)
    - Structural recursion (top-level - `Fixpoint`, local - `fix`)
    - lambda-functions (`fun ... => ...`)
  - Interactive proofs in intuitionistic logic
    - The usual logical quantifiers/connectives `forall, exists, ->, /\, \/, ~, <->`
    - Interactive tactics for proofs by induction, rewriting, etc.
- Not used: dependent types(!), co-induction, classical logic

Coq Examples.

- Inductive datatypes

```
Inductive nat: Set := O : nat | S : nat -> nat.
```

- Definitions by structural recursion

```
Fixpoint power (n m : nat) {struct m} : nat :=
  match m with
  | 0 => 1 | S m1 => n * power n m1
  end.
Eval compute in (power 2 5).
     = 32 : nat
```

- Partial evaluation

```
Eval cbv beta iota delta -[mult] in
  (fun n => power n 3).
= fun n : nat => n * (n * (n * 1)) :nat -> nat
```

Outline

Expression Sublanguage – Syntax.

- Data domain: simple binary trees (S-expressions with 1 atom)

```
Inductive Val: Set := | VNil: Val
  | VCons: Val -> Val -> Val | VBottom: Val.
```

- Expression language: tree constructors and selectors, identity, function composition, if-expressions

```
Inductive Selector: Set := | HD | TL.
Inductive Trm: Set := | Nil: Trm
  | Cons: Trm -> Trm -> Trm | Sel: Selector -> Trm
  | Id: Trm | Cmp: Trm -> Trm -> Trm
  | IfNil: Trm -> Trm -> Trm -> Trm | Bottom.
Infix "$" := Cmp (at level 60, right associativity).
Notation Hd := (Sel HD). Notation Tl := (Sel TL).
Infix "#" := Cons (at level 62, right associativity)
```

Expression Sublanguage – Semantics.

```
Definition evalSel (sel: Selector) (v: Val) : Val :=
  match v with | VCons v1 v2 =>
    match sel with | HD => v1 | TL => v2 end
  | _ => VBottom
  end.
Fixpoint evalT (t: Trm) (v: Val) {struct t} : Val :=
  match t with
  | Nil => VNil | Bottom => VBottom
  | Cons t1 t2 => VCons (evalT t1 v) (evalT t2 v)
  | Sel sel => evalSel sel v | Id => v
  | Cmp t1 t2 => evalT t1 (evalT t2 v)
  | IfNil t1 t2 t3 => match evalT t1 v with
    | VNil => evalT t2 v | VCons _ _ => evalT t3 v
    | VBottom => VBottom
    end
  end.
```

Simple Normalization.

- Based on simplifications like:
  - `Cmp Hd (Cons x y)` ≈ `x`
  - `IfNil (IfNil x y z) u v` ≈ `IfNil x (IfNil y u v) (IfNil z u v)`
- Produces terms in normal form:

```
Inductive NTrm: Set :=
  | NNil: NTrm | NCons: NTrm -> NTrm -> NTrm
  | NSelCmp: list Selector -> NTrm
  | NIfNil: list Selector -> NTrm -> NTrm -> NTrm
  | NBottom: NTrm.
```

- ... which can injected back into full-blown terms:

```
Fixpoint ntrm2trm (nt: NTrm) : Trm := ...
Definition evalNT nt v := evalT (ntrm2trm nt) v.
```

- Structurally-recursive implementation:

```
Fixpoint normConv (t: Trm) : NTrm := ...
```

Simple Normalization – Correctness.

- Tricky point - no full function composition in normal forms, yet we can still compose them:

```
Definition normNCmp : NTrm -> NTrm -> NTrm := ...
Lemma normNCmpPreservesEval: forall nt1 nt2 v,
evalNT (normNCmp nt1 nt2) v = evalNT nt1 (evalNT nt2 v).
```

- With the help of some other (simpler) lemmas like:

```
Lemma normSelsNCmpPreservesEvalT: forall sels nt v,
evalT (ntrm2trm (normSelsNCmp sels nt)) v
= evalSels sels (evalT (ntrm2trm nt) v).
Lemma normNCmpIfIf: forall sels1 sels2 nt1_1 nt1_2 nt2_1
nt2_2, let nt1 := NIfNil sels1 nt1_1 nt1_2 in
normNCmp nt1 (NIfNil sels2 nt2_1 nt2_2)
= NIfNil sels2 (normNCmp nt1 nt2_1) (normNCmp nt1 nt2_2).
```

- ... we can establish correctness of simple normalization:

```
Theorem normConvPreservesEval: forall t v,
  evalNT (normConv t) v = evalT t v.
```

Outline

Poor-man Explicit Substitutions.

- Primitives for pairing and function composition give us:
  - Variable-free programming
  - Simple form of explicit substitutions
- Example: `IfNil x1 x2 x3` has 3 free variables.
  - Pack them into an input tree: `x1 # x2 # x3`
  - Replace the original expression with:
    `IfNil Hd (Hd $ Tl) (Tl $ Tl)`
- Computing object-level representations of substitutions:
  `replaceAt (pos: list Selector) (t tr: NTrm): NTrm`
- Now, we can represent and apply the substitution of `Nil` for `x2` in the above expression:

```
let nt1 := normConv (IfNil Hd (Hd $ Tl) (Tl $ Tl)) in
let nt2 := normConv Nil in let x2p := TL::HD::nil in
ntrm2trm (normNCmp nt1 (replaceAt x2p (normConv Id) nt2)))
    = IfNil Hd Nil (Tl $ Tl)
```

Propagation of Test Outcomes in Branches.

- Internalize propagation of if-condition outcome:

```
Definition setNilAt (sels: list Selector): NTrm :=
  replaceAt sels (NSelCmp nil) NNil.
Definition setConsAt (sels: list Selector) : NTrm :=
  replaceAt sels (NSelCmp nil)
    (NCons (NSelCmp (sels ++ HD::nil))
           (NSelCmp (sels ++ TL::nil))).
Fixpoint propagateIfCond (nt: NTrm) {struct nt} : NTrm :=
  ...
  | NIfNil sels nt1 nt2 =>
    let nt1a := propagateIfCond nt1 in
    let nt2a := propagateIfCond nt2 in
    let nt1b := normNCmp nt1a (setNilAt sels) in
    let nt2b := normNCmp nt2a (setConsAt sels) in
    NIfNil sels nt1b nt2b
  ...
```

Propagation of Test Outcomes – Correctness.

- Test outcome propagation on top of simple normalization
  - Easier to give structurally recursive (total) definition
  - Easier to prove correctness on top of normalization correctness proof

```
Theorem propagateIfCondPreservesEval: forall nt v,
  evalNT (propagateIfCond nt) v = evalNT nt v.

Definition norm (t: Trm) :=
  propagateIfCond (normConv t).
Theorem normPreservesEval: forall t v,
  evalNT (norm t) v = evalT t v.
```

Outline

SWhile Language.

- Expression language - not Turing-complete
- Embed in simple imperative language (“SWhile”) with:
  - while-loops
  - single (implicit) variable

```
Inductive SWhileStmt: Set :=
  | Assign: Trm -> SWhileStmt
  | Seq: SWhileStmt -> SWhileStmt -> SWhileStmt
  | While: Trm -> SWhileStmt -> SWhileStmt.
Infix ";" := Seq (at level 65, right associativity).
Notation "'VAR' '<-' e" := (Assign e) (at level 64).
Notation "'WHILE' cond 'DO' body 'DONE'":=(While cond body)
```

- Further simplification - single while-loop (analog to Kleene normal forms in recursion theory)

```
VAR  <- initExp knf;
WHILE condExp knf DO VAR  <- bodyExp knf DONE;
VAR  <- finalExp knf
```

SWhile Language – Semantics.

- "SWhile" semantics in Coq?
  - inductive relations (elegant, non-executable)
  - or, a "folk" trick:
    - replace a partial function `f : X -> Y`
    - with a total function `f' : nat -> X -> option Y`, where
    - `f' d x = Some y -> f x = y` (`(f x)` is defined)
    - `f' d x = None` means `(f x)` cannot be computed in "stack depth" `d`
    - (`f'` is structurally recursive on `d`)
- Total "quasi-interpreter" for single-loop programs:

```
Definition evalKNF (d: nat) (knf: KNFProg) (v: Val)
  : option Val := ...
```

Loop Unrolling

- Analog to call unfolding in “SWhile”: loop unrolling
- Only a simple form of (single-step) unrolling considered – replace:

```
VAR  <- initExp knf;
WHILE condExp knf DO VAR  <- bodyExp knf DONE;
VAR  <- finalExp knf
```

- with:

```
VAR  <- ntrm2trm (norm
  (IfNil (condExp knf) Id (bodyExp knf) $ initExp knf));
WHILE condExp knf DO VAR  <- bodyExp knf DONE;
VAR  <- finalExp knf
```

- Process tree replaced by a stream of repeated unrollings

Final Supercompiler, Correctness.

- “Whistle” – the usual one: homeomorphic embedding
- No need for folding, generalization in this (over-)simplified setting
- Final supercompiler

```
Definition sscp ... (n : nat) (knf : KNFProg)
  : option KNFProg := ...
```

- Correctness: a) Totality (using Kruskal’s Tree Theorem as an axiom)

```
Theorem sscp_total: forall b knf, exists n,
  exists knf1, sscp b n knf = Some knf1.
```

- ... b) Preservation of semantics

```
Theorem sscp_correct: forall b knf knf1 n v1 v2,
  strictTrm (condExp knf) -> sscp b n knf = Some knf1 ->
  ((exists d1, evalKNF d1 knf v1 = Some v2) <->
   (exists d2, evalKNF d2 knf1 v1 = Some v2)).
```

Example of Supercompilation.

- Consider the usual Lisp-like encoding of lists and booleans as S-expressions (`WFalse := Nil`, `WTrue := Nil # Nil`, etc.)
- A program to check if the input list contains `WFalse`:

```
VAR  <- Id # WFalse; {VAR = input # output}
WHILE Hd DO
  VAR  <- IfNil (Hd $ Hd) (Nil # WTrue) (Tl $ Hd # Tl)
DONE;
VAR  <- Tl {VAR = output}
```

- Its specialized version – non-empty input list prepended with its negated head:

```
Definition listHasWFalse_knf_negdupHd :=
  let negate x := IfNil x WTrue WFalse in
  modifyKNFinput listHasWFalse_knf
    (IfNil Id Id (negate Hd # Id)).
```

Example of Supercompilation (cont.)

- Result of supercompiling the specialized version:

```
VAR <- IfNil Id (Nil # WFalse)
   (IfNil Hd (Nil # WTrue) (Nil # WTrue));
WHILE Hd DO
  VAR  <- IfNil (Hd $ Hd) (Nil # WTrue) (Tl $ Hd # Tl)
DONE; VAR  <- Tl
```

- ... and with superfluous `IfNil` removed further by hand:

```
VAR <- IfNil Id (Nil # WFalse) (Nil # WTrue);
WHILE Hd DO
  VAR  <- IfNil (Hd $ Hd) (Nil # WTrue) (Tl $ Hd # Tl)
DONE; VAR  <- Tl
```

- Loop still here but a simple static post-processing could remove it

Example of Supercompilation (end)

- We can define a downsized version of the supercompiler, without information propagation: `sscp'`
- Its result on the example:

```
VAR  <- IfNil Id
  (IfNil Id (IfNil Id Id (IfNil Hd (Nil # Nil) Nil # Id) # Nil)
     (IfNil Id (IfNil Hd (Nil # Nil # Nil) (IfNil Id Tl Id # Nil))
        (IfNil Hd (IfNil Id Tl Id # Nil) (Nil # Nil # Nil))))
  (IfNil Id (IfNil Hd (Nil # Nil # Nil) (IfNil Id Tl Id # Nil))
     (IfNil Hd (IfNil Id Tl Id # Nil) (Nil # Nil # Nil)));
WHILE Hd
DO VAR  <- IfNil (Hd $ Hd) (Nil # Nil # Nil) (Tl $ Hd # Tl) DONE;
VAR  <- Tl
```

Outline

Expression Language – Tests, Extensional Equivalence

- Normalization can simplify test generation

```
Inductive NTrm: Set :=
  | NNil: NTrm | NCons: NTrm -> NTrm -> NTrm
  | NSelCmp: list Selector -> NTrm
  | NIfNil: list Selector -> NTrm -> NTrm -> NTrm
  | NBottom: NTrm.
```

- Idea: expressions can extract information from input tree only through selector compositions
  - max. length of selector compositions = N
  - $\Rightarrow$ Expression cannot look deeper than N inside input tree
  - Trees of depth $\leq N$ should suffice as tests
- Finite tests sets $\Rightarrow$ extensional equivalence decidable

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More Realistic Language

- More powerful forms of loop unrolling?
- Add function calls to expression language

```
Inductive Trm: Set :=
  ...
  | Ref: FunRef -> Trm.
```

- It becomes Turing-complete
- Still possible to:
  - Isolate simple normalization, and information propagation
  - Implement them by structural recursion
- Complications:
  - How to specify semantics in Coq?
  - Normal forms - slightly more complicated
  - We need folding and generalization now
  - Termination proof of full supercompiler with generalization – more complicated(?)

Outline

Use Information Propagation in Isolation

- Apply (positive) information propagation in cases where we do need the full power of supercompilation (with its complications, like “whistle”, etc.)
- In systems like Coq itself; example:

```
Fixpoint listHasFalse (l: list bool) : bool :=
  match l with | nil => false
   | false::_ => true
   | true::l1 => listHasFalse l1
  end.
Goal forall b l, listHasFalse (b::negb b::l) = true.
  compute. fold listHasFalse.
...
 forall (b : bool) (l : list bool), (if b then
 if if b then false else true then listHasFalse l
 else true else true) = true
```

- Strengthen stream fusion?
- ...

Summary

- First formal verification of a supercompiler.
- Helped by a more fine-grained decomposition of the supercompilation process.
    - Structurally recursive deforestation and information propagation, with separate proofs.
    - Simple form of explicit substitutions also helpful.
- Outlook
    - Extend to more realistic languages, more powerful transformations.
    - Applications to test generation, compiler optimizations.
    - Some day: self-verifiable supercompiler