PROVING CORRECTNESS OF COMPILER OPTIMIZATIONS BY TEMPORAL LOGIC

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CLASSICAL COMPILER OPTIMIZATIONS

Most work by rewriting intermediate code. e.g., by the "Dragon book":

1. dead code elimination
2. constant folding
3. code motion and many more...

Widely used, a practical success. How do they work ? Pretty well.

What are their techniques?

- Data-flow lattices, usually greatest fixpoints
- "`gen`" and "`kill`" sets
- "definitions," "uses,", "transparency"
- forwards and backwards analyses
- worklist algorithms, etc etc etc

Alas: much arcane technology, with no semantic explanations!

THE QUESTION OF CORRECTNESS

Schmidt, Muchnick, Jones, others in the 1970's:

"Are these compiling analyses and transformations correct?"

Consequence: much thought and research.

1. **What is "correct" ? . . . must be "preserves program semantics".**
2. **What is "program semantics" ? Hmm. . .**
   We learn denotational, then operational semantics.
3. **Are the "data-flow based program analyses" correct ? And what does this mean, anyhow ? Hmm. . . we learn/invent abstract interpretation.**
4. **Are "compiling transformations" correct? . . . What was the question again?**

Alas, too many complex pieces to put together.

Year 2000 (about): Enter Oege De Moor, with a nifty transformational way to describe compiler optimizations.

REWRITE RULES FOR COMPILER OPTIMIZATIONS

$$\boxed{\mathcal{I} \Rightarrow \mathcal{I}' \text{ if } \phi}$$

- $\mathcal{I}, \mathcal{I}'$ are intermediate language instructions
- ϕ is a data+control flow property, expressed in a temporal logic.

Reading:

- If program π has instruction $\mathcal{I}$ at a control point p, and
- flow condition ϕ holds at p, then replace $stmt$ by $\mathcal{I}'$.

Program

$$\pi = \boxed{\ldots p : \mathcal{I} \ \ldots}$$

is rewritten into:

$$\pi' = \boxed{\ldots p : \mathcal{I}' \ldots}$$

A program-centric transformation description:

- p is "here" in the program. Enabling property.
- ϕ says "what I can/must see from here," looks forwards or backwards in the program's flow chart.

THE GOAL OF THIS WORK

A way to prove that such a transformation is correct.

Correctness:

IF program π' = the result of rewriting program π by applying this rule:

$$\boxed{\mathcal{I} \Rightarrow \mathcal{I}' \text{ if } \phi}$$

THEN π and π' have exactly the same semantics:

$$\boxed{[\![\pi]\!] = [\![\pi']\!]}$$

For example, they compute the same input-output function, and have the same termination properties.

MOTIVATION

- The use of either
  - rewrite rules to capture transformation or
  - temporal logic to prove correctness of compiler optimizations

  is not new (Boyle, Steffen, etc.). What's new:
- Temporal logic plays a central role in our framework, allowing much easier correctness proofs of classical optimizing transformations.
- The transformations are well-suited to automation. They can be (and have been) implemented in optimizing compilers. (De Moor et al)
- Specification and implementation come closer, increasing confidence in the compiler.
- Lerner, Millstein, Chambers have further automated our approach including proofs, using their D.S.L. Cobalt to express (weaker) rewrite conditions.

  (They also found many bugs in "obviously correct" rules – applicability conditions that needed strengthening to preserve semantics!)

MEANS

- Temporal logic can describe properties of state transition systems.
- The run-time state space of a program is a state transition system.
- Temporal side conditions (e.g., to apply a rewrite) relate to an anonymous "current state."
- Atomic propositions tell information about the current state.
- Future: Next-state quantifiers:
  AX means "for every next state,"
  EX means "for some next state" (branching-time, so we used CTL)
- Past: Previous-state quantifiers:
  $\overleftarrow{AX}$ means "for every previous state,"
  $\overleftarrow{EX}$ means "for some previous state"
- Path quantifiers refer to some or all computational futures or pasts.

DEAD CODE ELIMINATION

Goal: remove assignment statement that assigns a value that's never used:

$$\boxed{\mathsf{x} := \mathsf{e} \implies \texttt{skip}}$$

Side condition on the rewrite: The value assigned is never referenced again after the current (anonymous) program point.

The rewrite rule, with forward path quantifier A in its side condition ϕ:

$$\boxed{\mathsf{x} := \mathsf{e} \implies \texttt{skip if } A(\neg use(\mathsf{x}))}$$

A more liberal condition:

$$\boxed{\mathsf{x} := \mathsf{e} \implies \texttt{skip if } AX\ A(\neg use(\mathsf{x}))}$$

Use AX operator to start at the next point, since there's no harm if x is used in e.

A still more liberal condition with "weak until":

$$\boxed{\mathsf{x} := \mathsf{e} \implies \text{skip if } AX\ A(\ \neg use(\mathsf{x})\ W\ [def(\mathsf{x}) \wedge \neg use(\mathsf{x})]\ )}$$

PROGRAM SYNTAX

A program π has form:

$\pi =$ `read input;` $\mathcal{I}_1; \mathcal{I}_2;$ $\ldots$ $\mathcal{I}_{m-1};$ `write output;`

where

- $\mathcal{I}_1, \ldots \mathcal{I}_{m-1}$ **are instructions**
- **program labels:** $labels(\pi) = \{1, \ldots, m\}$
- **1 labels the first statement in π.**
- $m = exit(\pi)$ **labels the concluding** `write y;`

Grammar:

$$\mathcal{I} \in stmt \ ::= \ x := e \mid \texttt{skip} \mid \texttt{if } x \texttt{ goto } p_1 \texttt{ else } p_2$$

$$e \in \exp \ ::= \ c \mid x \mid \texttt{op}(e_1, \ldots, e_n)$$

$$\begin{array}{lll} x & \in & Variable \\ p_1, p_2 & \in & \{1, \ldots, labels(\pi)\}. \end{array}$$

OPERATIONAL SEMANTICS OF A PROGRAM

The semantic transition relation

$$\boxed{\rightarrow \subseteq PgmState \times PgmState}$$

for a program $\pi \in pgm$ is defined by:

1. **If $\mathcal{I}_t = \texttt{skip}$ then** $\boxed{(t, \sigma) \rightarrow (t+1, \sigma)}$
2. **If $\mathcal{I}_t = (\texttt{x} := e)$ then** $\boxed{(t, \sigma) \rightarrow (t+1, \sigma[\texttt{x} \mapsto [\![e]\!]\sigma])}$
3. **If $\mathcal{I}_t = (\texttt{if x goto } p_1 \texttt{ else } p_2)$ and $\sigma(\texttt{x}) = \texttt{true}$ then** $\boxed{(p, \sigma) \rightarrow (p_1, \sigma)}$
4. **If $\mathcal{I}_t = (\texttt{if x goto } p_1 \texttt{ else } p_2)$ and $\sigma(\texttt{x}) \neq \texttt{true}$ then** $\boxed{(p, \sigma) \rightarrow (p_2, \sigma)}$
5. $\boxed{(exit(\pi), \sigma) \rightarrow (exit(\pi), \sigma)}$

The initial state is:

$$initial_\pi(v) = (1, [\texttt{input} \mapsto v, y_1 \mapsto true, \ldots, y_k \mapsto true])$$

where $vars(\pi) \setminus \{\texttt{input}\} = \{y_1, \ldots, y_k\}$.

COMPUTATIONS

A computation prefix is a finite or infinite sequence $C \in PgmState^{*\omega}$:

$$C = \boxed{\pi, v \vdash (p_0, \sigma_0) \longrightarrow (p_1, \sigma_1) \longrightarrow \ldots}$$

such that

- $(p_0, \sigma_0) = initial_\pi(v)$ and
- $(p_i, \sigma_i) \longrightarrow (p_{i+1}, \sigma_{i+1})$ for all $i \geq 0$

The semantic function $[\![\pi]\!] : Value \longrightarrow Value$ is partial:

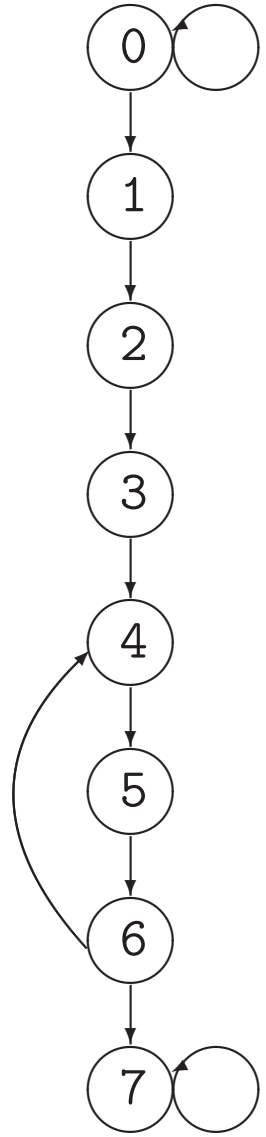
$[\![\pi]\!]v = \sigma_k(\mathrm{y})$ if there is a computation prefix

$$C = \boxed{\pi, v \vdash (p_0, \sigma_0) \longrightarrow \ldots \longrightarrow (p_k, \sigma_k)}$$

with $p_k = exit(\pi)$.

EXAMPLE PROGRAM AND CONTROL FLOW MODEL

```
     read x;

1:   five := 5;

2:   y := 0;

3:   c := five;

4:   y := y + c * x;

5:   x := x-1;

6:   if x then 4 else 7;

7:   write y;
```



(Loops at entry and exit for compatibility with temporal logic.)

CTL WITH FREE VARIABLES

In this talk: use an extended definition of CTL, CTL-FV to express conditions on the control flow. The extension is two-fold:

1. Reverse modality: Admits expressing backwards analyses, e.g., available expressions or constant propagation.

2. Propositions may have free variables, that are bound during model-checking to bits of the program being analyzed.

   - Implicit $\exists$ allows specification of rule schemata.
   - Explicit $\exists, \forall$ can be added (APLAS 2007).
   - OK as long as variable range is of bounded variation, e.g., bound to bits of or pointers to the program being analysed.

CTL WITH FREE VARIABLES

A CTL-FV model is a triple

$$\mathcal{M} = (S, \rightarrow, V)$$

where

1. S is a set of states
2. $\rightarrow \subseteq S \times S$ is a relation between states (forward and backward total).
3. The function $V : S \rightarrow 2^{AP}$ is called the valuation.

   It maps states to atomic propositions that hold in the given state.

A forward path has form

$$n_0 \rightarrow n_1 \rightarrow n_2 \rightarrow \ldots$$

A backward path has form

$$\ldots \rightarrow n_2 \rightarrow n_1 \rightarrow n_0$$

CONTROL FLOW MODEL FOR EXAMPLE PROGRAM

```
 read x;
1: five := 5;
2: y := 0;
3: c := five;
4: y := y + c * i;
5: x := x-1;
6: if x then 4 else 7;
7: write y;
```

Compiler-writers' jargon:

- X is "defined" if program contains `X := ...`
- X is "used" if program contains `_:= ...X...`

Transitions and atomic propositions:

S	$=$	$\{1, 2, 3, 4, 5, 6, 7\}$
$\rightarrow$	$=$	$\{1 \rightarrow 2, 2 \rightarrow 3, 3 \rightarrow 4, 4 \rightarrow 5, 5 \rightarrow 6, 6 \rightarrow 7, 6 \rightarrow 4, 7 \rightarrow 7\}$
$V(1)$	$\supseteq$	$\{\mathrm{node}(1), \mathrm{stmt}(\texttt{five := 5}), \mathrm{def}(\texttt{five}), \mathrm{conlit}(5)\}$
$V(2)$	$\supseteq$	$\{\mathrm{node}(2), \mathrm{stmt}(\texttt{y := 0}), \mathrm{def}(\texttt{y}), \mathrm{conlit}(0)\}$
$V(3)$	$\supseteq$	$\{\mathrm{node}(3), \mathrm{stmt}(\texttt{c := five}), \mathrm{def}(\texttt{c}), \mathrm{use}(\texttt{five})\}$
$V(4)$	$\supseteq$	$\{\mathrm{node}(4), \mathrm{stmt}(\texttt{y := y + c * i}), \mathrm{def}(\texttt{y}), \mathrm{use}(\texttt{y}), \mathrm{use}(\texttt{c}), \mathrm{use}(\texttt{i})\}$
$V(5)$	$\supseteq$	$\{\mathrm{node}(5), \mathrm{stmt}(\texttt{x := x - 1}), \mathrm{def}(\texttt{x}), \mathrm{use}(\texttt{x}), \mathrm{conlit}(1)\}$
$V(6)$	$\supseteq$	$\{\mathrm{node}(6), \mathrm{stmt}(\texttt{if x then 4 else 7}), \mathrm{use}(\texttt{x})\}$
$V(7)$	$\supseteq$	$\{\mathrm{node}(7), \mathrm{stmt}(\texttt{write y}), \mathrm{use}(\texttt{y})\}$

OPERATIONAL INTERPRETATION

A model checker will

- **not only find which nodes in a model satisfy a state formula, but also**
- **find instantiations of CTL variables that satisfy the formula.**

Technically: extend satisfaction relation $n \models \phi$ **to** $n \models_\theta \phi$. **Substitution** θ **binds the free variables of** ϕ**, so**

$$\boxed{n \models_\theta \phi \text{ holds for any } \theta \text{ such that } n \models \theta(\phi)}$$

The job of the model checker: given ϕ**, to return the set of all** n **and** θ **such that** $n \models_\theta \phi$.

For the example program and formula

$$\boxed{\phi = def(x) \wedge use(x)}$$

the model checker returns the following set:

$$\{\theta_1, \theta_2\} = \{[n \mapsto 4, x \mapsto \mathrm{y}], [n \mapsto 5, x \mapsto \mathrm{x}]\}$$

CTL-FV SYNTAX

CTL-FV formulas are either state or path formulas generated by the grammar with productions:

$$\phi ::= true \mid ap(\mathsf{x}_1, \ldots, \mathsf{x}_n) \mid \neg\phi \mid \phi_1 \wedge \phi_2$$

$$\begin{array}{lll} \mid & A\psi & \text{for all forward paths} \\ \mid & E\psi & \text{for some forward path} \\ \mid & \overleftarrow{A}\psi & \text{for all backward paths} \\ \mid & \overleftarrow{E}\psi & \text{for some backward paths} \end{array}$$

$$\begin{array}{lll} \psi ::= & X\phi & \\ \mid & \phi_1 \, U \phi_2 & \phi_1 \text{ holds until W} \\ \mid & \phi_1 W \, \phi_2 & \phi_1 \text{ holds weakly until W} \end{array}$$

$$\mathsf{x} \in \mathit{CTL\text{-}FV\text{-}variable}$$

SATISFACTION IN CTL-FV

Let $\mathcal{M}$ be a model. Then:

State Formulas:

$\mathcal{M}, n \models_\theta true$ **iff** $true$

$\mathcal{M}, n \models_\theta ap(x_1, \ldots, x_n)$ **iff** $ap(\theta x_1, \ldots, \theta x_n) \in V(n)$

$\mathcal{M}, n \models_\theta \neg\phi$ **iff** **not** $\mathcal{M}, n \models_\theta \phi$

$\mathcal{M}, n \models_\theta \phi_1 \wedge \phi_2$ **iff** $\mathcal{M}, n \models_\theta \phi_1$ **and** $\mathcal{M}, n \models_\theta \phi_2$

$\mathcal{M}, n \models_\theta E\ \psi$ **iff** $\exists path\ (n = n_0 \rightarrow n_1 \rightarrow \ldots) : \mathcal{M}, (n_i)_{i\geq 0} \models_\theta \psi$

$\mathcal{M}, n \models_\theta A\ \psi$ **iff** $\forall path\ (\ldots \rightarrow n_1 \rightarrow n_0 = n) : \mathcal{M}, (n_i)_{i\geq 0} \models_\theta \psi$

$\mathcal{M}, n \models_\theta \overleftarrow{E}\ \psi$ **iff** $\exists path\ (n = n_0 \rightarrow n_1 \rightarrow \ldots) : \mathcal{M}, (n_i)_{i\geq 0} \models_\theta \psi$

$\mathcal{M}, n \models_\theta \overleftarrow{A}\ \psi$ **iff** $\forall path\ (\ldots \rightarrow n_1 \rightarrow n_0 = n) : \mathcal{M}, (n_i)_{i\geq 0} \models_\theta \psi$

Path Formulas:

$\mathcal{M}, (n_i)_{i\geq 0} \models_\theta X(\phi)$ **iff** $\mathcal{M}, \models_\theta \phi$

$\mathcal{M}, (n_i)_{i\geq 0} \models_\theta \phi_1\ U\ \phi_2$ **iff** $\exists i \geq 0 : \mathcal{M}, n_i \models_\theta \phi_2 \wedge \forall 0 \leq j < i : \mathcal{M}, n_j \models_\theta \phi_1$

$\mathcal{M}, (n_i)_{i\geq 0} \models_\theta \phi_1\ W\ \phi_2$ **iff** $\exists i \geq 0 : \mathcal{M}, n_i \models_\theta \phi_2 \wedge$
$\forall 0 \leq j < i : \mathcal{M}, n_j \models_\theta \phi_1 \vee \quad (\forall k \geq 0 : n_k \models_\theta \phi_1 \wedge n_{k+1}$ **exists**$)$

REWRITING

A rewrite rule has form $\mathcal{I} \Rightarrow \mathcal{I}'$ if ϕ, where $\mathcal{I}$ and $\mathcal{I}'$ are instructions built from program and CTL variables, and ϕ is a CTL-FV formula.

Define

$$Apply(\pi, \pi', n, \mathcal{I} \Rightarrow \mathcal{I}' \text{ if } \phi)$$

to be true if and only if for some substitution θ, the following hold:

1. $\mathcal{M}, n \models_\theta stmt(\mathcal{I}) \wedge \phi$

2. $\pi =$ `read input; 1: ... n: θ(I); ... write output;`

3. $\pi' =$ `read input; 1: ... n: θ(I'); ... write output;`

where $1 \leq n \leq m + 1$ and the $\ldots$'s are unchanged.

DEAD CODE ELIMINATION

Goal: **remove an assignment statement that assigns a value that is never used:**

$$\mathsf{x{:=}e} \quad \Longrightarrow \quad \texttt{skip}$$

Side condition on the rewrite: **the value assigned is never referenced again before it is (re-)defined.**

The rewrite rule with its side condition is written

$\mathsf{x{:=}e} \quad \Longrightarrow \quad \texttt{skip}$
if
$AX\ A(\ \neg use(\mathsf{x})\, W\, def(\mathsf{x}) \wedge \neg use(\mathsf{x})\ )$

CONSTANT FOLDING

Constant folding transformation replaces a variable reference with a constant value:

$$\mathbf{x{:=}y} \quad \Longrightarrow \quad \mathbf{x{:=}c}.$$

To check: whether all assignments to y assign it the same constant value.

To check this condition we use the past temporal operators as follows:

$$\mathbf{x{:=}y} \quad \Longrightarrow \quad \mathbf{x{:=}c}$$

if

$$\overleftarrow{A}\,(\ \neg \mathtt{def}(\mathbf{y}) \wedge \neg \mathtt{stmt}(\mathbf{read}\ \_)$$
$$U\ \mathit{stmt}(\mathbf{y{:=}c}) \wedge \mathit{conlit}(\mathbf{c}))$$

Point: to ensure that all paths

- from program entry "read "
- to instruction x:=y

contain a definition y:=c. By definition of $\models_\theta$ all the c's must be the same.

REWRITING AT MORE THAN ONE POINT

We may specify several rewrites and side conditions at once.

Example: To transform two nodes the form of the rewrite would be:

$$
\begin{array}{l}
\quad n : \mathcal{I}_1 \Longrightarrow \mathcal{I}'_1 \\
\quad m : \mathcal{I}_2 \Longrightarrow \mathcal{I}'_2 \\
\text{if} \\
\quad n \models \phi_1 \\
\quad m \models \phi_2
\end{array}
$$

Operational interpretation: find a substitution θ that satisfies

$$n \models_\theta stmt(\mathcal{I}_1) \wedge \phi_1 \text{ and } m \models_\theta stmt(\mathcal{I}_2) \wedge \phi_2$$

and use θ to alter the program in places n and m.

CODE MOTION/LOOP INVARIANT HOISTING

A double rewrite:

p : `skip` $\Longrightarrow$ **x:=e**

q : **x:=e** $\Longrightarrow$ `skip`

if

$p \models A(\neg use(\mathbf{x})\ W\ node(q))$

$q \models \neg use(\mathbf{x}) \wedge$

$\overleftarrow{A}((\neg def(\mathbf{x}) \vee node(q)) \wedge$

$trans(\mathbf{e}) \wedge \neg \mathtt{stmt}(\textbf{read x})\ W\ node(p))$

Example: **lift** `x:=a+b;` **from label** 3 **to label** 1.

```
1:  skip;
2:  if...then 3 else 6;
3:  x := a + b;        (inside a loop)
4:  y := y − 1;
5:  if y then 3 else 6;
6:  x := 0;
```

Optimized code: **lift** `x:=a+b;` **from label** $q = 3$ **to label** $p = 1$.

A METHOD FOR SEMANTIC EQUIVALENCE

To show: $Apply(\pi, \pi', p, \mathcal{I} \Rightarrow \mathcal{I}' \text{ if } \phi)$ **implies** $[\![\pi]\!] = [\![\pi']\!]$, **i.e..** $\forall$ **input** v

$initial_\pi(v) \rightarrow^* (exit, \sigma)$	**for program** π **if and only if**
$initial_{\pi'}(v) \rightarrow^* (exit, \sigma')$	**for program** π'**, and** $\sigma(\texttt{output}) = \sigma'(\texttt{output})$

The problem: how to link the
- **temporal** property ϕ ("futures" and "pasts") to
- the transformation $I \Longrightarrow I'$.

Solution: enrich the semantics and its transition system to **computation prefixes**:

$$C = \boxed{\pi, v \vdash s_0 \rightarrow \ldots \rightarrow s_t}$$

Prefix transition system $\mathcal{T}_{pfx}(\pi, v)$:
Define

$$C \rightarrow C_1 \in \mathcal{T}_{pfx}(\pi, v)$$

if and only if

$$C_1 = \boxed{\pi, v \vdash s_0 \rightarrow \ldots \rightarrow s_t \rightarrow s_{t+1}}$$

PROOF METHOD

Programs π and π' are semantically equivalent: $[\![\pi]\!] = [\![\pi']\!]$ if there exists a bisimulation relation $\mathcal{R}$ on computation prefixes, such that for all v:

1. **(Base Case) $\mathcal{R}$ holds between the initial computation prefixes, i.e.,**

$$\boxed{((\pi, v \vdash \mathit{initial}_{\pi}(v)) \;\; \mathcal{R} \;\; (\pi', v \vdash \mathit{initial}_{\pi'}(v)))}$$

2. **(Step) If $C_1 \;\mathcal{R}\; C_1'$ and $C_1 \longrightarrow C_2$ and $C_1' \longrightarrow C_2'$ then $C_2 \;\; \mathcal{R} \;\; C_2'$.**
3. **(Equivalence) If**

$$\boxed{\begin{array}{l} C \;\mathcal{R}\; C' \text{ and} \\ C = \pi, v \vdash s_0 \longrightarrow s_1 \ldots \longrightarrow (p_t, \sigma) \text{ and} \\ C' = \pi', v \vdash s_0' \longrightarrow s_1' \ldots \longrightarrow (p_{t'}, \sigma') \end{array}}$$

then

$$\boxed{\begin{array}{l} (i) \;\; p_t = \mathit{exit}(\pi) \iff p_{t'}' = \mathit{exit}(\pi') \text{ and} \\ (ii) \;\; p_t = \mathit{exit}(\pi) \wedge p_{t'}' = \mathit{exit}(\pi') \\ \qquad \Rightarrow \sigma_t(\mathtt{output}) = \sigma_{t'}'(\mathtt{output}) \end{array}}$$

DEAD CODE ELIMINATION

$$\phi \equiv AX\ A(\ \neg use(\mathsf{x}) W\, def(\mathsf{x}) \wedge \neg use(\mathsf{x})\ )$$

Consider $C \in \mathcal{T}_{pfx}(\pi, v)$ and $C' \in \mathcal{T}_{pfx}(\pi', v)$ such that:

$$\begin{array}{l} C \ = \ \pi, v \vdash s_0 \longrightarrow s_1 \longrightarrow \ldots \longrightarrow s_t \ , \\ C' \ = \ \pi', v \vdash s'_0 \longrightarrow s'_1 \longrightarrow \ldots \longrightarrow s'_r \end{array}$$

and

$\forall 0 \leq i \leq t : s_i = (p_i, \sigma_i)$ **and**
$\forall 0 \leq i \leq r : s'_i = (p_i, \sigma'_i)$.

Define $C \mathcal{R} C'$ if and only if $t = r$ and for any $0 \leq i \leq t$:

1. $p_i = p'_i$
2. $\forall j < i : p_j \neq p \Rightarrow \sigma_i = \sigma'_i$ **and**
3. $\exists j < i : p_j = p \wedge \sigma_i \backslash \mathrm{x} = \sigma'_i \backslash \mathrm{x}$

The bisimulation conditions are easily verified.

CONSTANT FOLDING

$$\mathbf{x{:=}y} \quad \Longrightarrow \quad \mathbf{x{:=}c}$$

if

$$\overleftarrow{A}\,(\ \neg\mathtt{def}(\mathbf{y}) \wedge \neg\mathtt{stmt}(\mathbf{read\ x})\ U\ \mathit{stmt}(\mathbf{y{:=}c}) \wedge \mathit{conlit}(\mathbf{c}))$$

In this case relation $\mathcal{R}$ is the identity relation(!)

That is, we wish to prove that for any length n, a computation prefix of π of length n is equal to a computation prefix of π' with the same length.

Statement and proof of conditions: **slightly more complex.**

CODE MOTION/LOOP INVARIANT HOISTING

p : skip $\Longrightarrow$ x:=e

q : x:=e $\Longrightarrow$ skip

if

$p \models A(\neg use(\mathsf{x})\ W\ node(q))$

$q \models \neg use(\mathsf{x}) \wedge$

$\overleftarrow{A}((\neg def(\mathsf{x}) \vee node(q)) \wedge$

$trans(\mathsf{e}) \wedge \neg \mathtt{stmt}(\textbf{read x})\ W\ node(p))$

Suppose

$$C = \pi, v \vdash (p_0, \sigma_1) \rightarrow \ldots \rightarrow (p_t, \sigma_t)$$
$$C' = \pi', v \vdash (p'_0, \sigma'_1) \rightarrow \ldots \rightarrow (p'_{t'}, \sigma'_{t'})$$

Define the $\mathcal{R}$ relation as: $C\mathcal{R}C'$ if and only if $t = t'$, $p_i = p'_i$ for all $0 \leq i \leq t$ and one of the following cases holds:

1. $\sigma_t = \sigma'_t \ \wedge \ \forall 0 \leq i < t : p_i \notin \{p, q\}$
2. $\sigma_t = \sigma'_t \ \wedge \ \exists 0 \leq i < t : p_i = q \ \wedge \ \sigma_i = \sigma'_i \wedge \forall i < j < t : p_j \notin \{p, q\}$
3. $\exists 0 \leq i < t : p_i = p \ \wedge \ (\sigma_t \setminus \mathbf{x} = \sigma'_t \setminus \mathbf{x}) \wedge (\sigma_i \setminus \mathbf{x} = \sigma'_i \setminus \mathbf{x}) \ \wedge \ \forall i < j < t : p_j \notin \{p, q\}$

EXCERPT FROM THE END OF THE PROOF

(No, you're not morally obliged to read and understand the details!) Suppose

$$C = \pi, v \vdash (p_0, \sigma_1) \longrightarrow \ldots \longrightarrow (p_t, \sigma_t)$$
$$C' = \pi', v \vdash (p'_0, \sigma'_1) \longrightarrow \ldots \longrightarrow (p'_{t'}, \sigma'_{t'})$$

Assumptions:

	π		π'
$p_i = p$	`skip`	$\Rightarrow$	`x:=e`
$p_t = q$	`x:=e`	$\Rightarrow$	`skip`

and $C \mathcal{R}_3 C'$. Then x maps to the same value in σ_{t+1} and σ'_{t+1}:

$$\begin{aligned}
\sigma_{t+1}(\mathsf{x}) &= [\![\mathsf{e}]\!]\sigma_t && \text{(semantics of } \mathcal{I}_{p_t} = (\mathrm{x}{:=}\mathrm{e})) \\
&= [\![\mathsf{e}]\!]\sigma_{i+1} && \text{(argument in paper)} \\
&= [\![\mathsf{e}]\!]\sigma_i && \text{(semantics of } \mathcal{I}_{p_i} = \mathtt{skip}) \\
&= [\![\mathsf{e}]\!](\sigma_i \setminus \mathsf{x}) && \text{(since } \mathrm{x} \notin \mathit{vars}(\mathrm{e})) \\
&= [\![\mathsf{e}]\!](\sigma'_i \setminus \mathsf{x}) && \text{(since } C \mathcal{R}_3 C' \Rightarrow \sigma_i \setminus \mathrm{x} = \sigma'_i \setminus \mathrm{x}) \\
&= [\![\mathsf{e}]\!](\sigma'_i) && \text{(since } \mathrm{x} \notin \mathit{vars}(\mathrm{e})) \\
&= \sigma'_{i+1}(\mathsf{x}) && \text{(semantics of } \mathcal{I}'_{p_i} = (\mathrm{x}{:=}\mathrm{e})) \\
&= \sigma'_t(\mathsf{x}) && \text{(argument in paper)} \\
&= \sigma'_{t+1}(\mathsf{x}) && \text{(semantics of } \mathcal{I}'_{p_t} = \mathtt{skip})
\end{aligned}$$

Thus $\sigma_{t+1} = \sigma'_{t+1}$.

SUMMARY

- We specified transformations as rewrite rules with temporal logic formulas as side conditions.
- We set up a framework in which temporal logic plays a central role in correctness proofs of classical compiler optimizing transformations.
- Describing transformations as rewrite rules is not new; but use of temporal logic to specify when to apply them, and to prove their validity, seems to be new.

To prove correctness: show that a transformation does not change semantics, i.e., $[\![\pi]\!] = [\![\pi']\!]$.

The only creative part is finding the relation $\mathcal{R}$.

This relation is closely related to the temporal logic side conditions of the transformation. The remainder of the proof is usually straightforward.

IMPERATIVE PROGR. TRANSF. BY REWRITING

Part of a larger project (David Lacey, Eric Van Wyk, Oege de Moor) to study

- declarative methods of specifying optimizations and
- means of automatically generating optimizers from these specifications.

Specifications of transformations are rewrite rules with temporal logic side conditions, implemented by a graph rewriting system and model checker.

Very simple programming language so far.

- Adding new statement types does not affect the effectiveness of our method.
- Exceptions and procedures would, however, require changes to the control flow model and the transition systems used in the proof.
- The specification of the transformations however does not dramatically change.

FUTURE WORK

- Systematic derivation **of the bisimulation relation** $\mathcal{R}$ **from rewrite rule:** $\mathcal{I} \Rightarrow \mathcal{I}'$ if ϕ
- **Is it** decidable **whether rewrite rule** $\mathcal{I} \Rightarrow \mathcal{I}'$ if ϕ **is valid, i.e., preserves semantics of all programs ?**
- **Is it possible to derive the weakest side condition, given a rewrite rule?**
- **What is the** efficiency **of program transformation by such rule-based rewriting ? Can it be engineered to be of** comparable efficiency **with traditional compiler optimizations?**
- **Improved specification language.**
- **Conjecture: This can be applied to supercompilation (cf. Geoff, Gavin)**

FUTURE WORK II

Our language: rather like a traditional compiler's "intermediate language".

The method can validate many traditional optimizing compiler transformations, e.g. in

- the "Dragon book" or Muchnick's compendium.

Lerner, Millstein and Chambers saw the January 2002 POPL paper. By November 2002 they had designed

- a stronger compiler intermediate language with pointers and procedures
- a language "Cobalt" to specify optimization rewrite rules (a domain-specific language, weaker than the CTL that we use)
- Implemented their strategy with Simplify, an automatic theorem prover
- Found lots of subtle bugs in enabling conditions

Since then: Rhodium, inferring optimiser flow functions from semantics.

RELATED WORK

- **J. Knoop, O. Rüthing, B. Steffen.** Optimal Code Motion: Theory and Practice. **ACM TOPLAS, 16(4):1117–1155, 1994.**
- **D.A. Schmidt, B. Steffen.** Program analysis as model checking of abstract interpretations. **Proc. 5th SAS, 1998.**
- **D. Kozen and M. Patron.** Certification of compiler optimizations using Kleene algebra with tests. **In CL2000, LNAI vol. 1861, Springer-Verlag,, 2000, pp. 568–582.**
- **D. Lacey and O. de Moor.** Imperative program transformation by rewriting. **In Compiler Construction, vol. 1113 of LNCS, pp. 52–68. Springer-Verlag, 2001.**
- **S. Lerner, T. Millstein, C. Chambers.** Automatically proving the correctness of compiler optimizations. **ACM PLDI, pp. 220-231, 2003.**
- **Lerner et al** Automatic Inference of Optimizer Flow Functions from Semantic Meanings. **PLDI, 2007.**